PART 13  Unsaturated zone

Capillary effects

\[ R = \text{radius of curvature} \]

\[ \Theta < 90^\circ \rightarrow \text{wetting fluid} \]

\[ \Theta > 90^\circ \rightarrow \text{non-wetting fluid} \]

\[ \Theta = 90^\circ \rightarrow \text{no capillary forces} \]

\[ r/R = \cos \Theta \]

Capillary fringe depends on:
- tube diameter
- surface tension (interface tension)
- wetting properties (of the solid)

Meniscus forms because:

(1) Water molecules “adhere” to walls of the tube (“wetting”)

(2) water molecules on the surface of the water (and air-water interface) have imbalanced charge, so they cling more tightly to each other, form a “net” which supports the weight of the water below.
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\[ P_{\text{air}} - P_{\text{water}} = (2\sigma \cos \Theta)/r \]

where \( \sigma \) = interfacial tension (force per length, e.g., dyn/cm, N/m, etc.)

If we set \( P_{\text{air}} = 0 \) (gauge pressure), then

\[ P_{\text{water}} = P = -(2\sigma \cos \Theta)/r = -2\sigma/R \]

This means that pressure at the interface is negative.

\[ \psi = h_c = -P/\rho g = (-2\sigma/R)\rho g = \text{pressure head} \]

This also means that the pressure head is negative (because of negative pressure)

Total head:

\[ h = z + \psi = h_c + P/\rho g = h_c - h_c = 0 \]

Therefore, total head is zero.

**Notes on surface tension:**

Common units: dyn/cm and N/m

\( \text{dyne (dyn)} = \text{g}\cdot\text{cm/s}^2 \)
\( \text{N} = \text{kg}\cdot\text{m/s}^2 \)

dyne and newton are units of force.
dyn/cm and N/m are units of surface tension

Unit conversion:

1 dyn = 10\(^{-5}\) N
1 dyn/cm = 10\(^{-3}\) N/m

Surface tension of distilled water: 72 dyn/cm = 72\(\times\)10\(^{-3}\) N/m
Application to soil

The radius of curvature (R) is small in dry soils and in fine soils. This leads to more negative pressures, i.e., soil has more suction.

Conclusions:

Dry soils have greater negative pressure (more suction) than wet soils.

Fine soils have greater negative pressure (more suction) than coarse soils.

Storage in unsaturated media

\[ \Theta = \frac{V_{\text{water}}}{V_{\text{soil}}} \]

\[ \Theta_r = \text{specific retention} \]

\[ S_y = \text{specific yield} \]

\[ \Theta_r + S_y = n \]

Specific retention - when water films are disconnected.
Relationship between water content and pressure

Definitions:

\[ \eta = \text{porosity} = \frac{V_{\text{void}}}{V_{\text{total}}} \]

\[ \Theta = \text{moisture content} = \frac{V_{\text{water}}}{V_{\text{total}}} \leq \eta \]

\[ S = \text{degree of saturation} = \frac{V_{\text{water}}}{V_{\text{void}}} = \frac{\Theta}{\eta} \leq 1 \]

Head is: \[ h = z + \frac{p}{\rho g} = z + \psi \]

Pressure head \( \psi \) depends on soil properties and on moisture content:

\[ \psi = \psi(\text{matrix, } \Theta) \]

The relationship between moisture content and pore pressure is called \( \psi - \Theta \) curve or pore pressure - saturation curve (figure on the right) or water retention curve.

Note that the curve has hysteretic behavior, i.e., the curve depends on the wetting history of the sample.

This relationship must be determined empirically for each sample of interest. Experimental data are collected and a function is fitted. This function may be just a fitting function (without any physical meaning) or it may involve parameters that have a physical meaning.

An example of a fitting function is:

\[ \psi = \frac{a(\Theta_s - \Theta)^b}{\Theta^c} \]

where \( \psi \) is pressure, \( a, b \) and \( c \) are fitting parameters, \( \Theta \) is water content, and \( \Theta_s \) is water content at saturation (often taken as equal to porosity).
Another example is the **van Genuchten** relationship:

\[
\Theta = \Theta_r + \frac{\Theta_s - \Theta_r}{[1 + (\alpha \psi)^n]^m} 
\]

or

\[
\frac{\Theta - \Theta_r}{\Theta_s - \Theta_r} = \frac{1}{[1 + (\alpha \psi)^n]^m} 
\]

where \(\Theta_r\) is minimum water content (at retention), \(\Theta_s\) is water content at saturation (i.e., porosity), and \(\alpha\), \(m\) and \(n\) are fitting constants.

**Things to note in the van Genuchten equation:**

1. Fitting constant \(n\) is not porosity, but a free fitting parameter;

2. Quantity \((\Theta - \Theta_r)/ (\Theta_s - \Theta_r)\) is normalized water content; it assumes values between 0 (for water content \(\Theta_r\)) and 1 (for water content \(\Theta_s\)).

**Reference:**

Hydraulic conductivity of unsaturated media

Wet $K >$ dry $K$ because water molecules can choose shorter path in interconnected pores than in disconnected pores of dry media.

$K$ is a function of moisture content $\Theta$ (or saturation $S$): $K$ decreases as $\Theta$ decreases (nonlinear function).

Because pore pressure is a function of moisture content, $K$ is also a function of pressure (or suction): $K$ decreases with decreasing pressure (or with increasing negative pressure, i.e., with increasing suction).

An example of fitting function is:

$$K(\psi) = \frac{a}{b + \psi^m}$$

where $a$, $b$ and $m$ are fitting parameters.

Another example:

$$K(\Theta) = K_{sat} \Theta^n + c$$

where $K_{sat}$ is the saturated hydraulic conductivity, $n$ is the porosity, and $c$ is an empirical constant.

Note: At low saturation, clay has higher $K$ than sand has!
Measuring unsaturated hydraulic conductivity

HyProp device - small scale, laboratory

Experimental results in the graph below are for two (of 36 total) samples from Manitou Experimental Forest, Colorado. Next page shows the derivation of hydraulic conductivity as a function of water content, and the following page shows all 36 samples.
The first graph is the measured water retention curve for two Manitou samples (the same graph as on the previous page). The second graph is the derived function of hydraulic conductivity vs. pressure. The last graph is a combination of the first two, and shows the unsaturated hydraulic conductivity as a function of water content.
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Manitou, Colorado, USA

log[$\text{Hydraulic conductivity (cm/d)}$]

Soil moisture (m$^3$ m$^{-3}$)
Measuring unsaturated hydraulic conductivity

COSMOS probe - large scale, field

Information about COSMOS probe: cosmos.hwr.arizona.edu and publications listed therein.
Comparison of laboratory data (HyProp) and field data (COSMOS).
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\[ K_F, K_C \]

Pressure Head (cm-H₂O)

Hydraulic Conductivity (cm/sec)

- \( K_F \) - fine dune sand
- \( K_C \) - (coarser) medium blasting sand

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Figure 16  Hydraulic conductivity of materials in the sand box in Figure 15.
Equations of unsaturated flow

Continuity:

\[-\nabla \cdot \mathbf{q} = \frac{\partial \Theta}{\partial t}\]

Flow:

\[\nabla \cdot [K(\Theta)\nabla (\psi + z)] = \frac{\partial \Theta}{\partial t}\]

\[\psi = \psi(\Theta) \text{ and } K = K(\Theta)\]

are necessary to solve the equation.

Things to note:

(1)

If \(\Theta = \text{const}\) and uniform and medium uniform

\[\nabla \cdot [K(\Theta)\nabla (z + \psi)] = \nabla K(\Theta)\nabla (z + \psi) + K(\Theta)\nabla^2 (z + \psi)\]

\[\nabla K(\Theta) = 0 \text{ because } \Theta = \text{const}\] (i.e., \(K = \text{const}\))

\[\nabla \psi = 0 \text{ for the same reasons}\]

Thus, the equation simplifies to:

\[K(\Theta)\nabla^2 z = \frac{\partial \Theta}{\partial t}\]

\[\nabla z = \bar{k} \text{ (unit gradient)}\]

The gravity is the only driving force for flow, i.e., we have vertical flow. Horizontal flow is impossible if \(\Theta = \text{const}\).

(2)

It also follows that

\[\nabla^2 z = \nabla (\nabla z) = 0, \text{ so it must be true that } \frac{\partial \Theta}{\partial t} = 0\]

It means that it must be steady flow if \(\Theta = \text{const}\).
Heterogeneous materials

Because $\psi_{\text{sand}} > \psi_{\text{clay}}$, flow will be from sand to clay.

The flow will continue so that the pressure head in sand ($\psi_{\text{sand}}$) and that in clay ($\psi_{\text{clay}}$) become equal.

Because sand has lower $K$, flowlines at the boundary between clay and sand are diverging. This is different than under saturated conditions, where flow lines were converging.