PART 18  

Salt-water intrusion

In coastal areas salt water is in contact with fresh ground water. An interface between them can be described using different formulations. Two of them are described below.

**Ghyben-Herzberg relation**

Consider a simple interface between salt water and fresh water (figure). The interface between the two can be described using the Ghyben-Herzberg equation.

In equilibrium (no flow) the pressures at the interface are equal: \( p_s = p_f \)

\[
p_s = z_s \gamma_s = z_s \rho_s g
\]

\[
p_f = (z_s + z_f) \gamma_f = (z_s + z_f) \rho_f g
\]

Equating them, we get:

\[
z_s \rho_s g = (z_s + z_f) \rho_f g
\]
and solving for $z_s$, we get:

$$z_s = \frac{\rho_f}{\rho_s - \rho_f} \cdot z_f$$  \text{ Ghyben-Herzberg relation}

Look at water densities:

- sea water: $\rho_s = 1.025$ g/cm$^3$
- fresh water: $\rho_f = 1.000$ g/cm$^3$

Put them in the Ghyben-Herzberg relation:

$$z_s = \frac{1}{(1.025-1.000)} z_f = 40 z_f$$

Thus, there is 40 times more fresh water below the mean sea level than above it. In other words, for every 1 m of water table elevation above sea level there is 40 m of fresh water below it.

This relation assumes a sharp boundary between salt and fresh water and no dispersion. It also assumes that fresh water forms a wedge into sea water, and that fresh water discharges into the ocean at a single point - an impossibility. Other relationships take these into account.

**Glover relation**

We now realize that fresh water discharges into the sea over an area rather than along a line (as was the case in Ghyben-Herzberg) and that vertical component of flow is not negligible as water moves along the interface (see figure).
Glover developed the following equation for the shape of the freshwater-saltwater interface:

\[ z^2 = \frac{2 \cdot Q \cdot x \cdot \rho_f}{(\rho_s - \rho_f) \cdot K} + \left( \frac{Q \cdot \rho_f}{(\rho_s - \rho_f) \cdot K} \right)^2 \]

Glover equation

where: 
\( Q \) = flow in aquifer per unit length of shoreline;  
\( K \) = hydraulic conductivity of aquifer;  
\( x, z \) = coordinate distances from shoreline (figure).

Using the density of salt water of 1.025 g/cm\(^3\) and that of freshwater of 1 g/cm\(^3\) and substituting \( z=0 \) into the Glover equation, we compute the width \( W \) of the zone through which fresh water flows into the sea:

\[ W = \frac{Q \cdot \rho_f}{2(\rho_s - \rho_f) \cdot K} \]

By substituting \( x=0 \) in the Glover equation, we can compute the depth \( z_0 \) of the freshwater-saltwater interface beneath the shoreline:

\[ z_0 = \frac{Q \cdot \rho_f}{(\rho_s - \rho_f) \cdot K} \]
Pumping of coastal aquifer

Pumping results in declining water table (cone of depression). Because for every meter of drop of the water table the salt water will rise 40 m (see Ghyben-Herzberg equation), the depth to freshwater-saltwater interface will decrease fast and so will the volume of freshwater in the aquifer.

Thus, pumping near a coast must be designed carefully so that the depth to the saltwater-freshwater interface be preserved. One way of doing so is by using injection wells installed between the shoreline and the pumping wells (left figure). Injected water will push the interface towards the sea. Some injected water will be lost to the sea, but no sea water will be allowed to flow past the barrier. Often multiple wells are arranged along a line, forming a gallery of wells (right figure).