

Groundwater Modeling

Laboratory 4

HWR 431/531

Lab 4- Groundwater Modeling

Introduction:

Several methods exist for solving groundwater flow problems. For simple geometries analytical solutions suffice. Complex problems may be difficult to solve analytically and so other methods are needed. Graphical solutions and analog models have been used extensively for a wide variety of problems, including those that involve heterogeneity and the vadose zone. The development of fast computers has allowed numerical models to become the most common method of solving groundwater flow problems. In this laboratory, you will explore and evaluate these different methods of solving groundwater flow problems.

Background:

Analytical Solutions:

An analytical solution is one that may be derived from the principles of groundwater flow. Consider the following diagram adapted from Freeze & Cherry (1979; Fig.2.25):

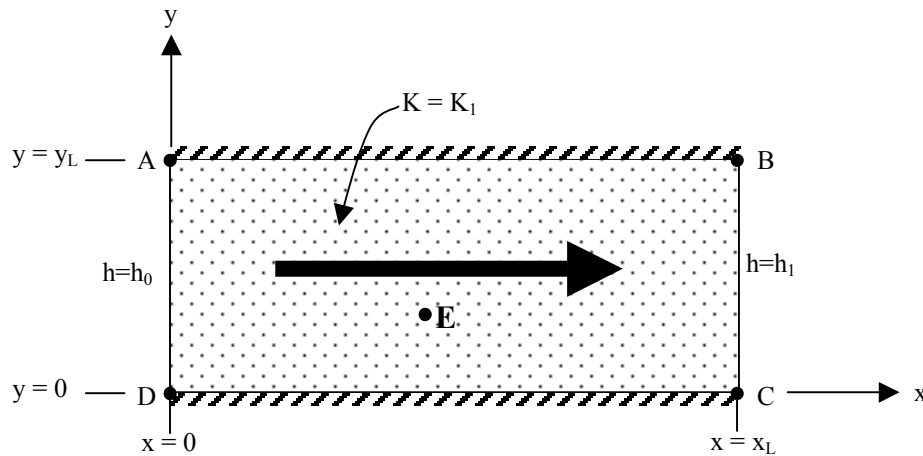


Figure 1 – A simple flow system. AD & BC are constant head boundaries and AB & DC are impermeable boundaries.

The physical setup of the system (i.e. geometry, hydraulic conductivity and location of prescribed head boundaries) are the defining characteristics of the flow domain. The following analytical solution for

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steady state flow defines the head at any point, E, within the domain (see Appendix 3, Freeze & Cherry, 1979):

$$h(x) = h_o - (h_o - h_1) \frac{x}{x_L} \quad (1)$$

It is evident that this solution is unique only if the heads at the boundaries, h_o & h_1 , are known. If the boundary conditions are changed, the heads within the flow system will change accordingly.

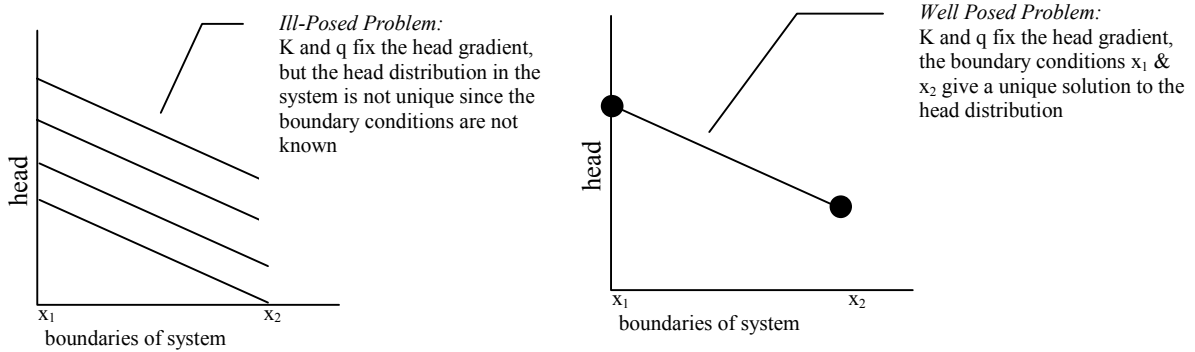


Figure 2 – The absolute value of head cannot be determined in a groundwater system unless the problem is well posed.

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Graphical Solutions:

(see Ch. 5 of Freeze & Cherry, 1979)

The graphical representation of the head (i.e. equipotential lines) and flow direction (i.e. flowlines) is called a *flow net*. Groundwater flow problems may be solved quickly and easily (with practice) by sketching flow nets. Although they do not always offer the precision of other methods, flow nets do provide the opportunity to gain insight into complex systems with relatively little effort and are thus excellent conceptual tools for preliminary work.

Several assumptions underlie flow net construction:

- 1) Two dimensional flow. This implies no recharge or discharge perpendicular to the plan of the flow net.
- 2) Isotropic medium. Hydraulic properties (e.g. hydraulic conductivity) are independent of the direction of measurement.
- 3) Steady-state flow. At a point, velocity of flow does not vary in magnitude or direction.
- 4) Fixed physical characteristics of fluids. This implies constant temperature and density of the water.
- 5) Single phase flow.
- 6) Flow obeys Darcy's Law.

NOTE: Some of the above assumptions are necessary only for simple types of flow nets. Special techniques can be used to adjust for anisotropy, 3-D flow, etc.

If these assumptions are met, the following rules should be followed to sketch the flow net:

- 1) All flowlines are perpendicular to equipotential lines.
- 2) Impermeable boundaries are flow lines.
- 3) Flowlines terminate perpendicular to constant head boundaries.
- 4) Flowlines do not intersect except at stagnation points and singular points (eg. wells).
- 5) Equipotential lines do not close except around sinks and sources.
- 6) Equipotential surfaces refract into horizontal surfaces at the water table.
- 7) No two equipotential lines can intersect.
- 8) No equipotential lines can terminate except upon a boundary of the field of flow.
- 9) All squares must be *curvilinear*. (A circle will be tangent to all 4 sides.)
- 10) Partial streamtubes are acceptable near boundaries.

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Given these rules, we can construct a flow net for the system in Figure 1. If all of the above rules are to apply, we can immediately see that the flow net must consist of straight equipotential and flowlines (Fig.3).

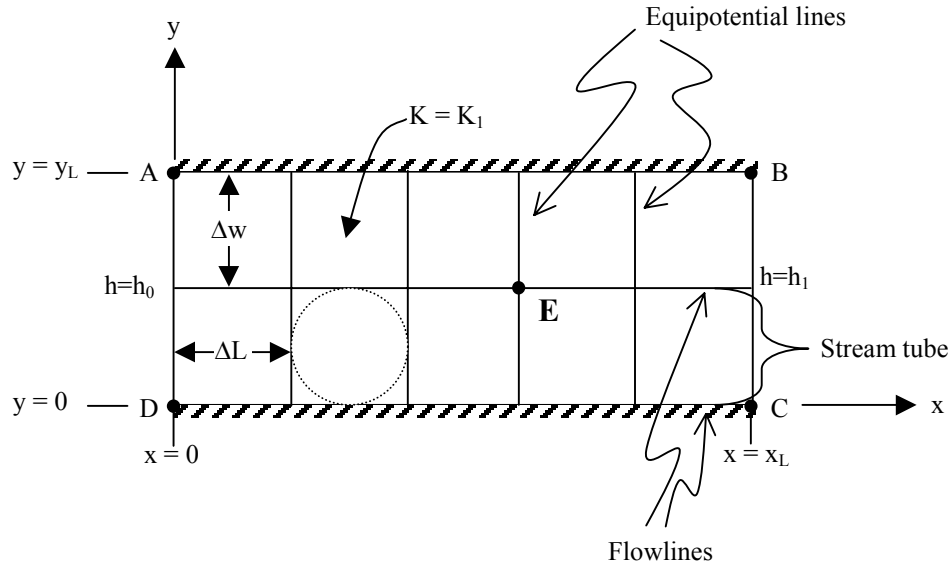


Figure 3 – Flow net for a simple flow system. Notice that all squares formed by equipotential and flowlines are curvilinear. (AD & BC are constant head boundaries and AB & DC are impermeable boundaries.)

Flow nets can be used to perform some simple calculations. If the flow net is constructed with equal head drops between equipotential lines, Darcy's law can be expressed as:

$$Q = nKb\Delta w \frac{\Delta h}{\Delta L} \quad (2)$$

where:

Q = total discharge [L^3]

n = number of streamtubes (i.e. area between two adjacent flowlines)

K = hydraulic conductivity [LT^{-1}]

b = thickness perpendicular to flow net (i.e. into the page) [L]

Δw = width of streamtube [L]

ΔL = distance between *head drops* [L]

Δh = head drop *between* equipotential lines [L]

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If the flow net is constructed such that $\Delta w = \Delta L$ (i.e. curvilinear) and recalling that $Kb = T$ (transmissivity), then the equation simply becomes:

$$Q = nT\Delta h \quad (3)$$

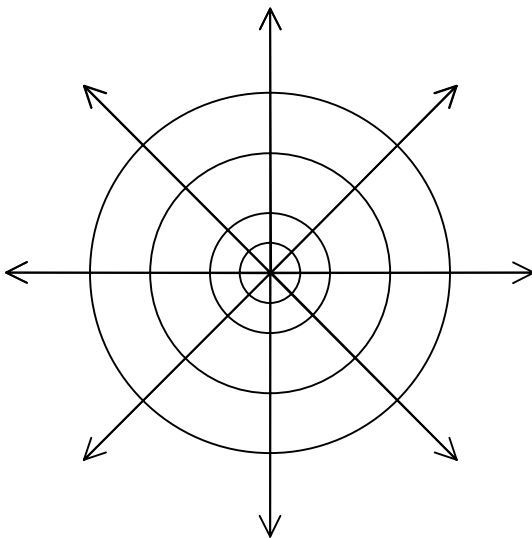
Fractional values of n and Δh are allowed along flow boundaries.

Travel time can be estimated along a stream tube by the following equation:

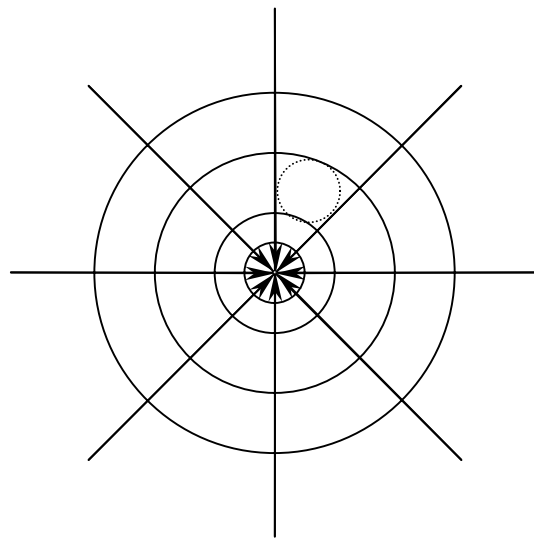
$$\text{travel time} = \sum_0^i \Delta t = \frac{n_o}{K\Delta h} \sum_0^i (\Delta L)^2 \quad (4)$$

where $\Sigma \Delta t$ is the sum of individual travel times across each distance ΔL in the flow net and n_o is the effective porosity. To obtain the best estimate of total travel time, each Δt should be calculated individually along the flow path.

Examples of flow nets:

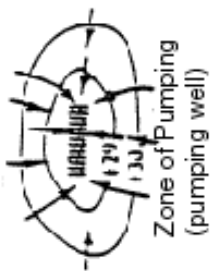


Zone of recharge (i.e. injection well)

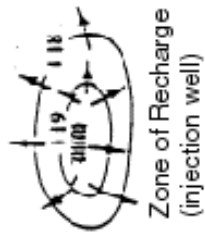


Zone of pumping

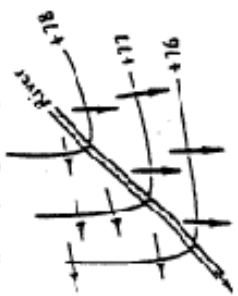
Flow Nets: Examples



Zone of Pumping (pumping well)



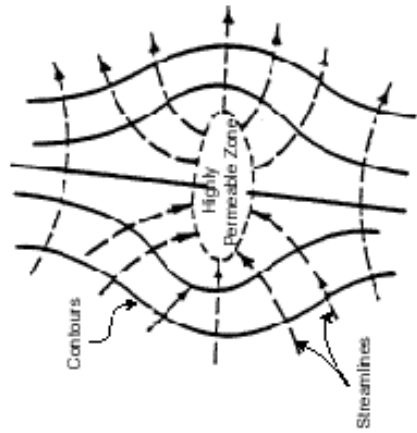
Zone of Recharge (injection well)



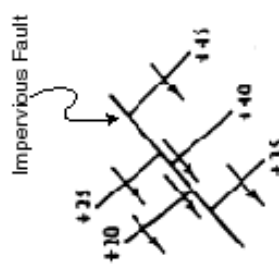
Losing Stream (stream is above water table)



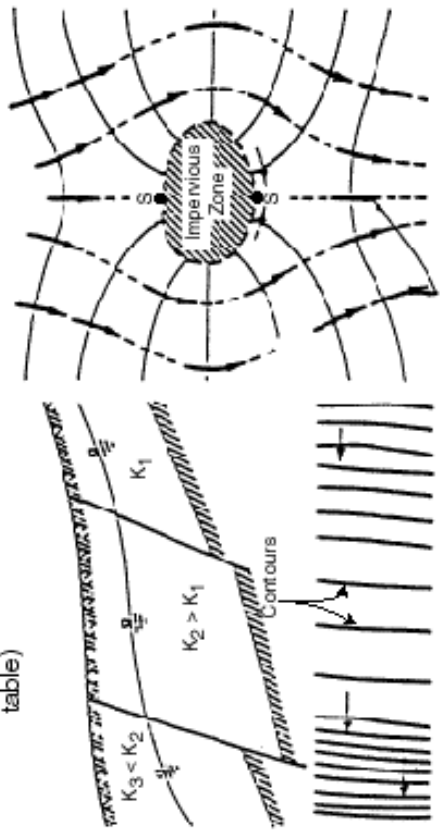
Gaining Stream (recharge area)



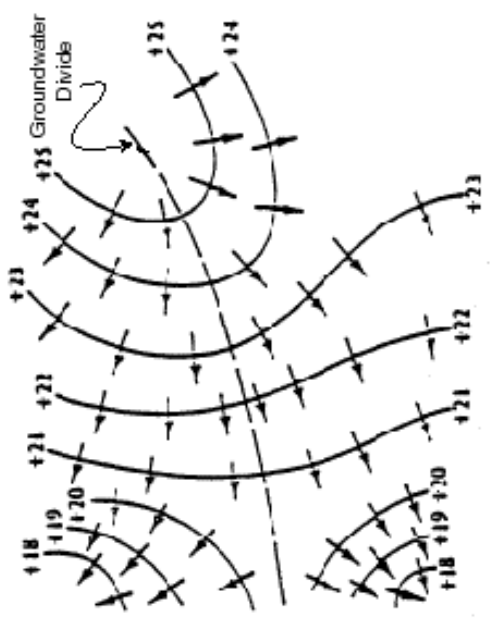
Effect of a Highly Permeable Zone (i.e. a lake within a larger flow system)



Difference in water levels across an impervious fault



Effect of Variable Transmissivity



A Groundwater Divide

Effect of Impervious Zone ● Stagnation Point

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Electrical Analog Models:

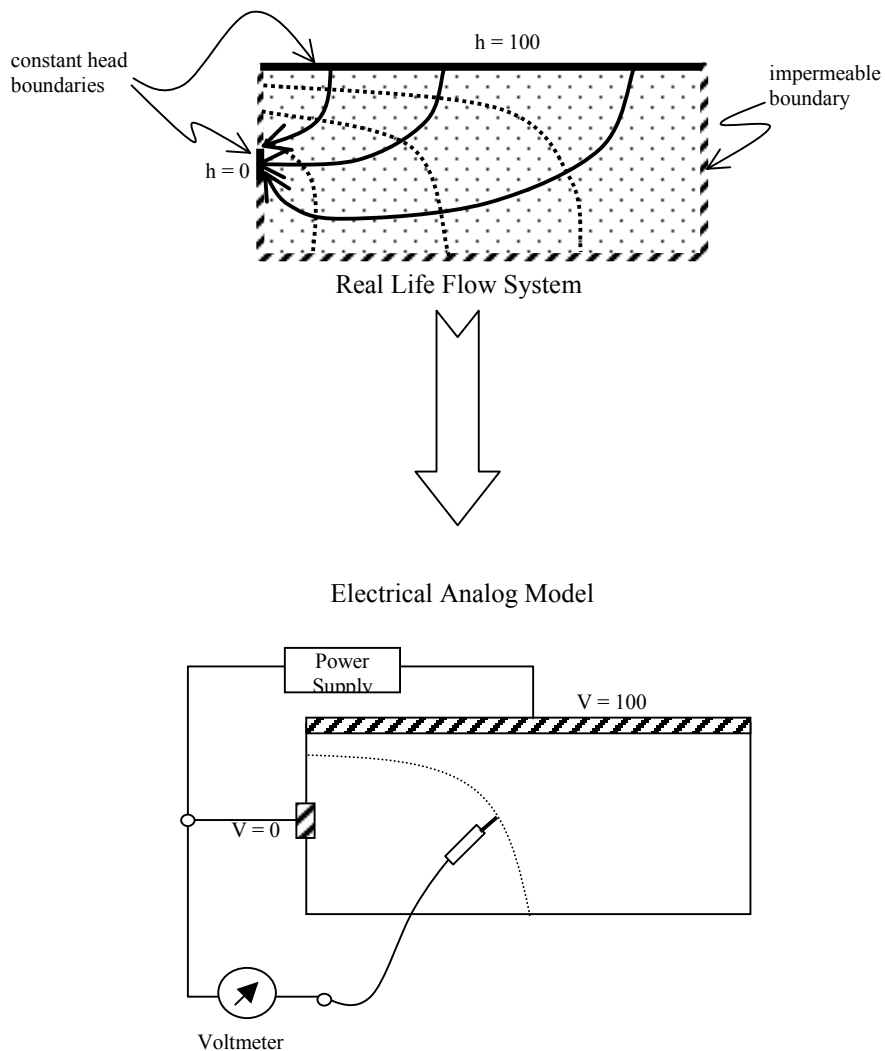
Before the advent of computer simulation, a very common tool for predicting groundwater movement was the use of electrical analog models. Under steady state conditions the two dimensional movement of groundwater, electricity and heat are governed by Laplace's equation (Eq. 5):

$$\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2} = 0 \quad (5)$$

where: x, y represent the primary coordinate axis
 G represents the quantity of interest (ex. head, voltage or temperature)

Using conductive-paper one may create an electrical system that has an analogous response to boundary conditions as an aquifer.

Figure 5 – Electrical Analog Simulation (after Freeze & Cherry, 1979)



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Given this electrical model, the following properties are analogous:

head (h) \rightarrow voltage (V)

flux (q) \rightarrow current density (I)

hydraulic conductivity (K) \rightarrow specific electrical conductivity (σ)

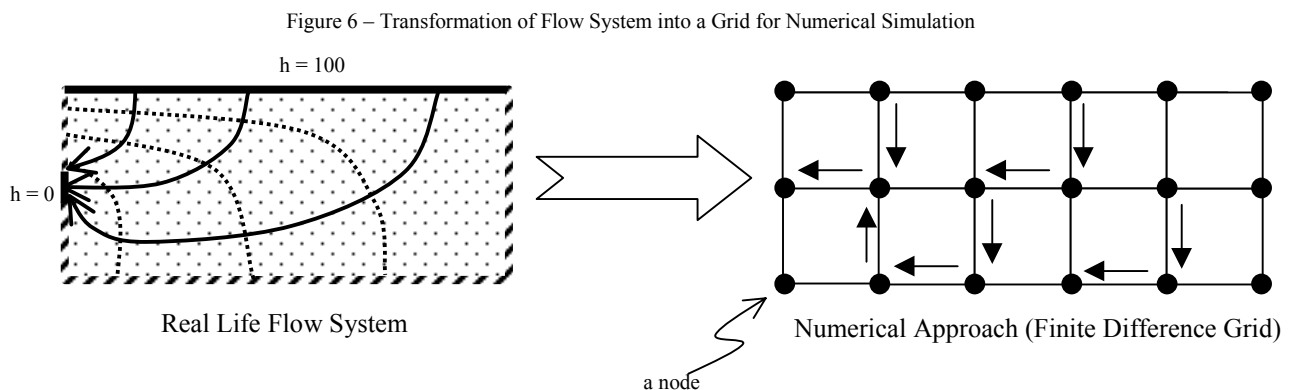
The conductive paper can be cut to represent the shape of any two dimensional basin. The hydrologic properties of the basin are controlled by the properties of the paper; areas where the hydraulic conductivity is high can be represented in the model by conductive paint. Lining edges with conductive paint (and/or a wire) connected to a power source of a given voltage simulates prescribed head boundaries. The head at any point in the basin can be read directly using a voltmeter.

Any two-dimensional basin subject to steady state flow can quickly and accurately be modeled using this method, making it is a powerful tool.

Numerical Models:

Numerical models of groundwater flow can be used to solve complex groundwater flow problems such as those that have irregular boundaries or complicated geological configurations. These models are based on the discretization of the flow field into a grid. At each intersection of gridlines exists a **node**. Equations such as Darcy's Law can be applied between each of the nodes to calculate unknown head values. Sections 5.3 & 8.7 in Freeze and Cherry (1979) provide a good introduction to this procedure.

These sections are required reading for this exercise.



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Laboratory Assignment:

This lab is divided into two weeks. The first week will focus on flow nets and electrical analog models. The second week will take place in the computer lab where you will design and run several flow models using Visual MODFLOW.

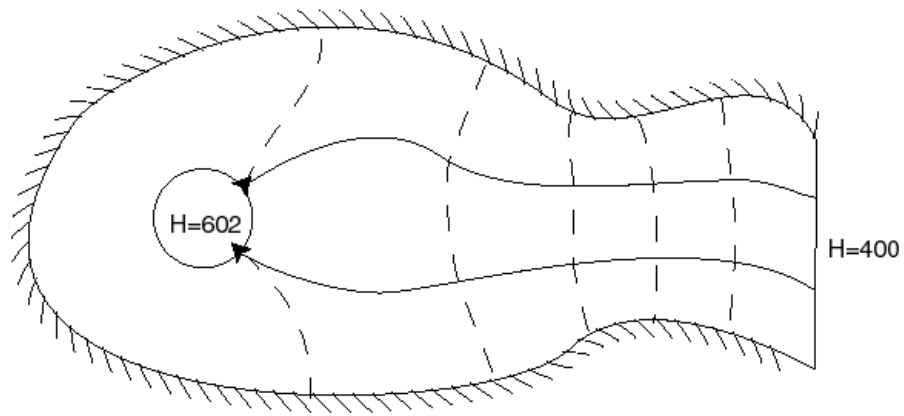
Week 1 – Flow Nets & Electrical Analog Models:

There are nine questions for you to answer on the following pages. These should help you to understand the basic concepts behind the construction of flow nets. Lab time is allotted for you to begin this assignment but it should be completed at home. You are also to complete flow nets for two electrical analog models. This activity must be completed during the lab period.

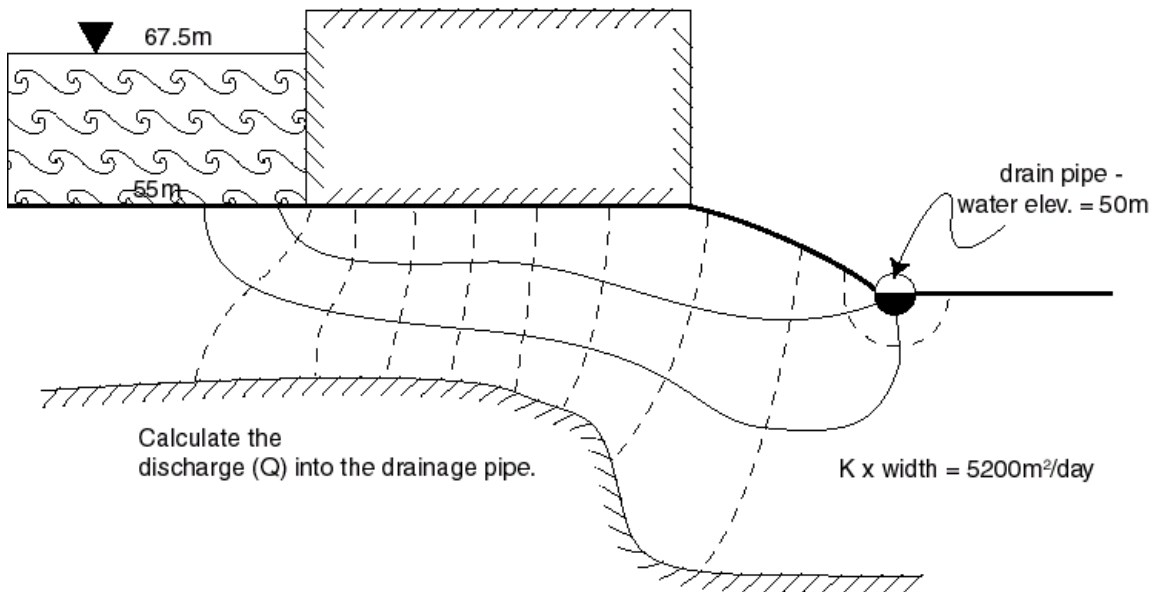
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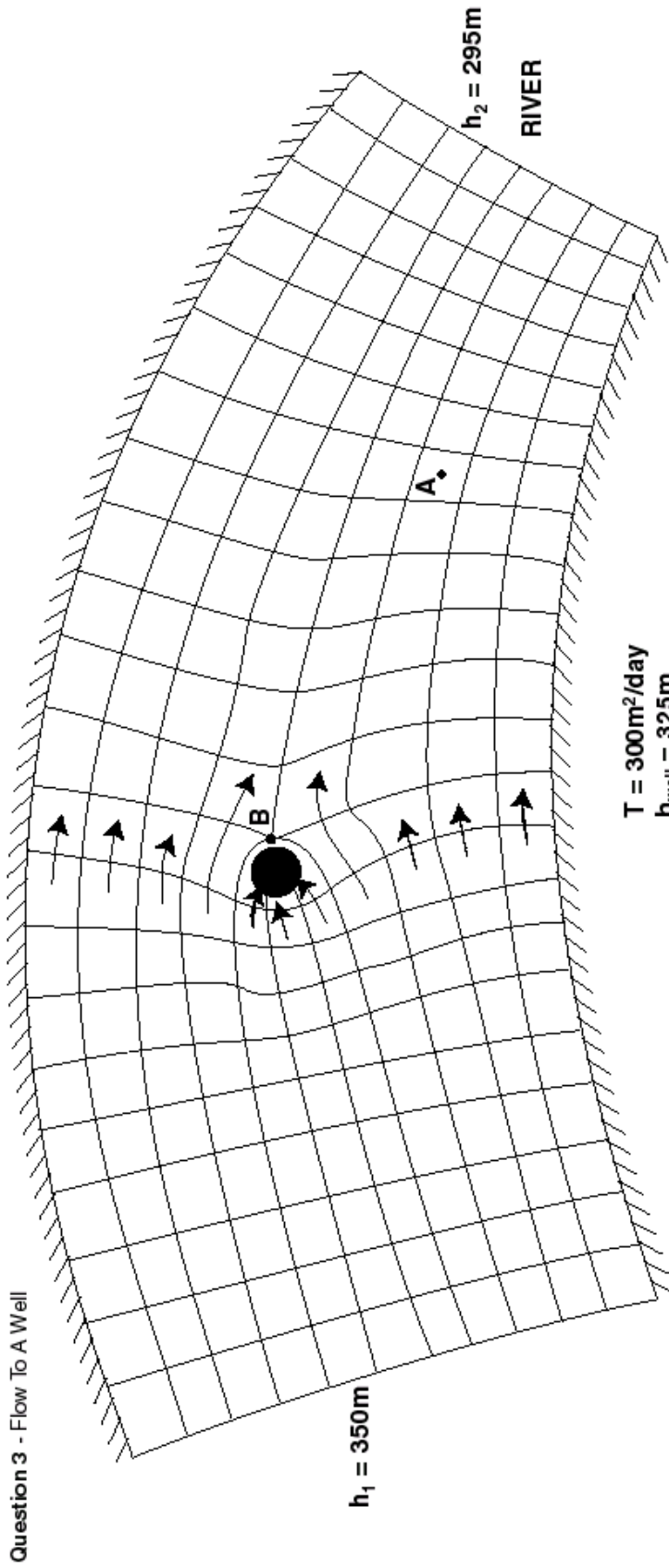
Question 1

What is wrong in this picture? Circle and name at least 5 different things wrong with this flownet.



Question 2



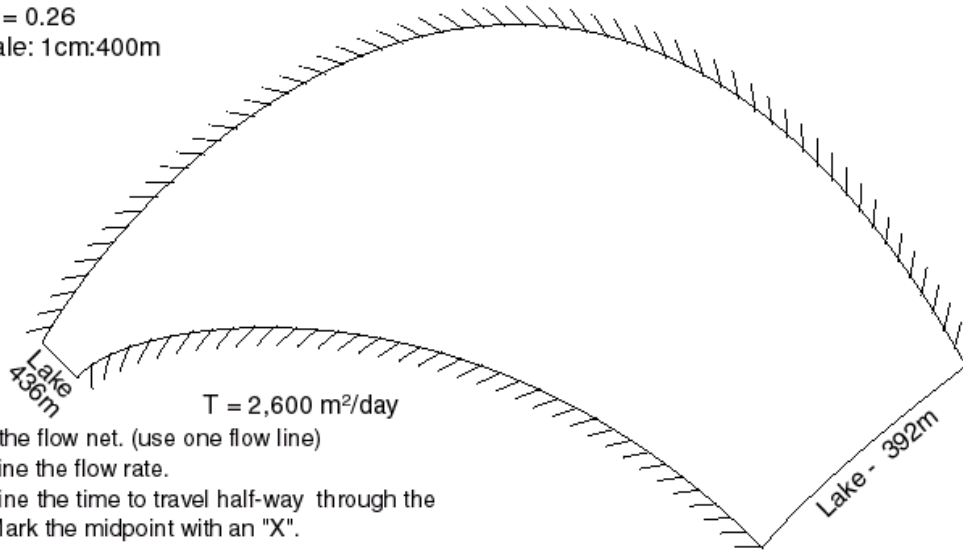


- A. Calculate the flow to the well in m^3/day .
- B. Calculate the flow to the river in m^3/day .
- C. Find the head at point A.

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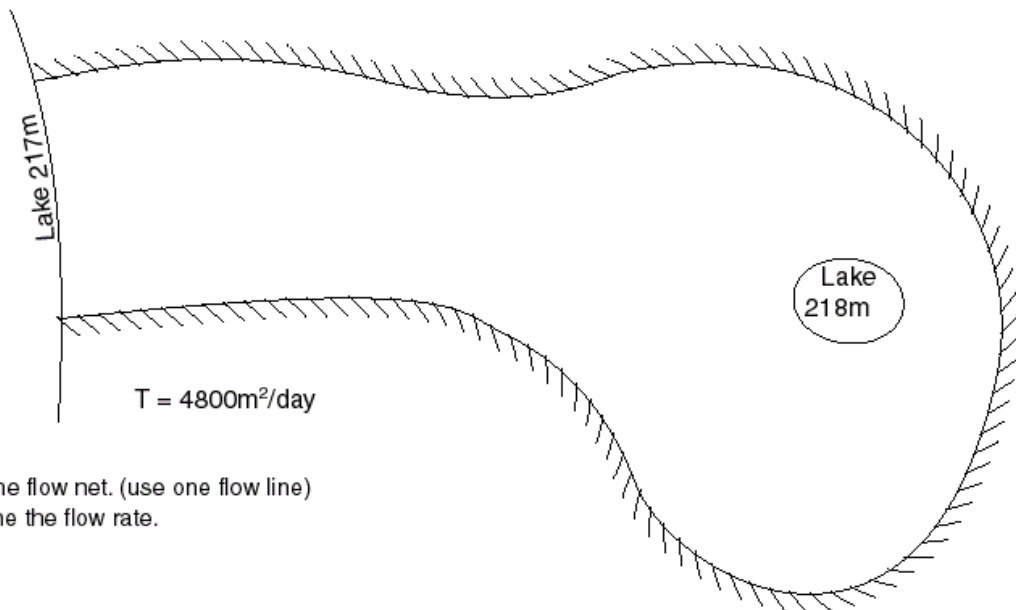
Question 4

$b = 100\text{m}$
 $n_e = 0.26$
scale: 1cm:400m



- Sketch the flow net. (use one flow line)
- Determine the flow rate.
- Determine the time to travel half-way through the system. Mark the midpoint with an "X".

Question 5



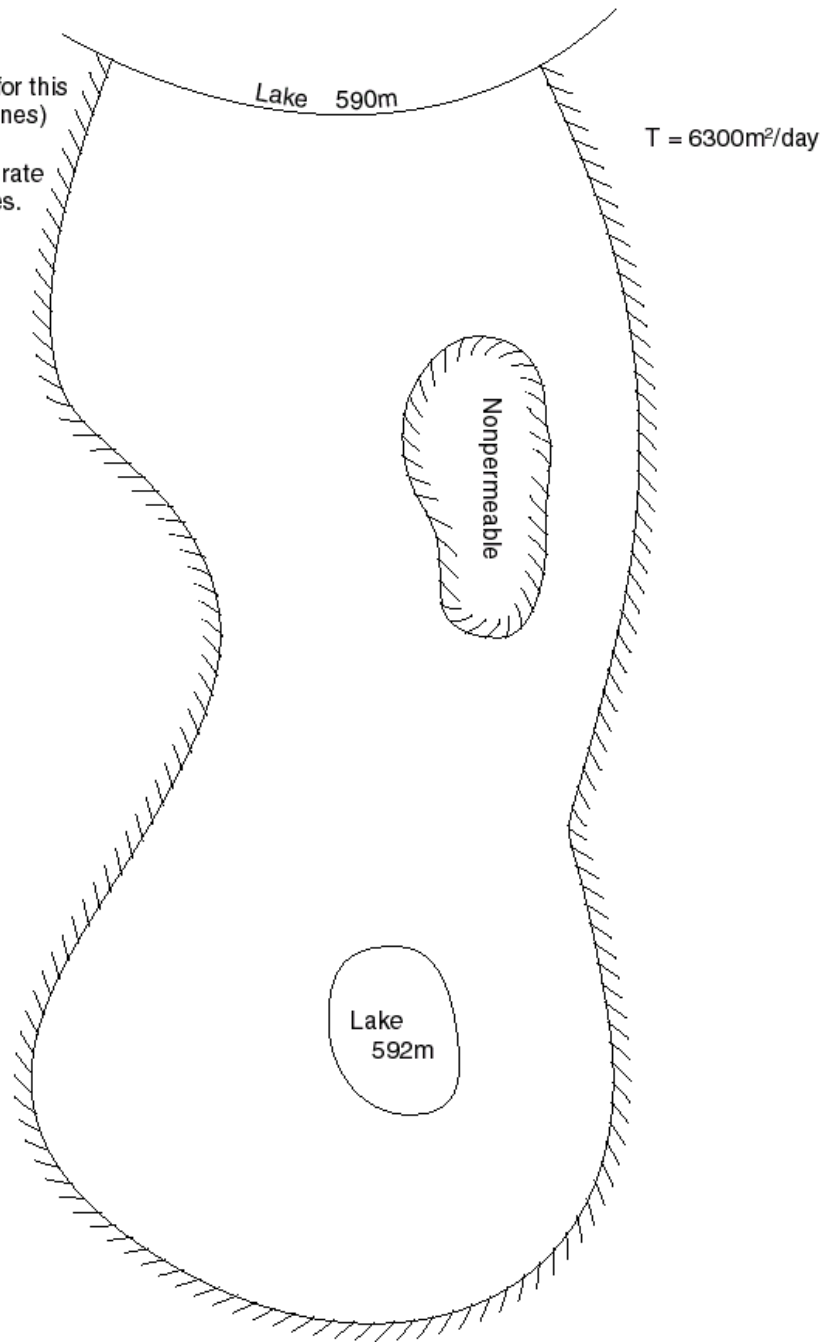
- Sketch the flow net. (use one flow line)
- Determine the flow rate.

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Question 6

A. Sketch a flow net for this system. (use 2 flow lines)

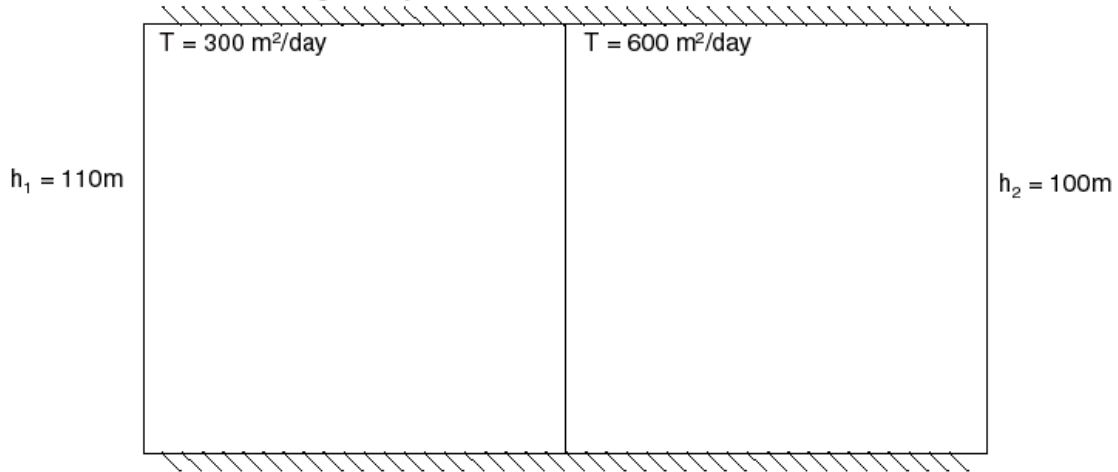
B. Calculate the flow rate between the two lakes.



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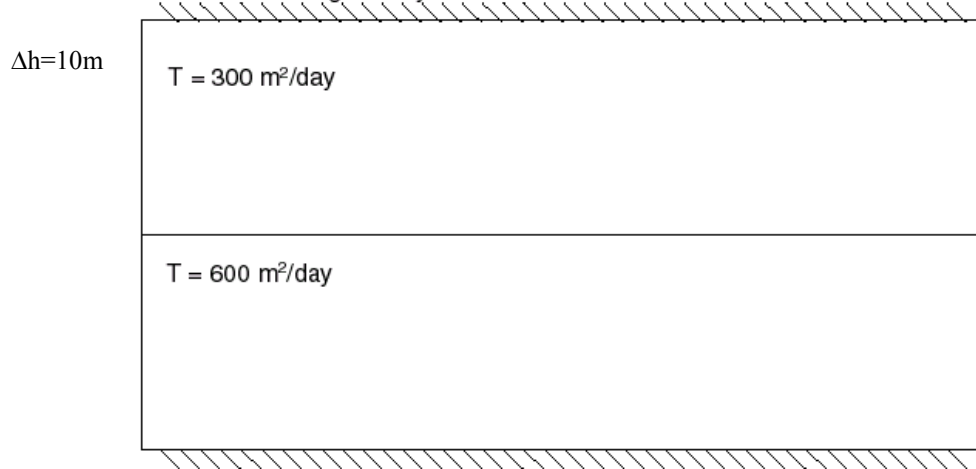
Question 7

- A. Sketch a flow net for the following system (use 1 flow line/keep Q the same for both flow tubes)
B. Calculate the flow rate through the system.



Question 8

- A. Sketch a flow net for the following system (use 2 flow line/keep Q the same for all flow tubes)
B. Calculate the flow rate through the system.



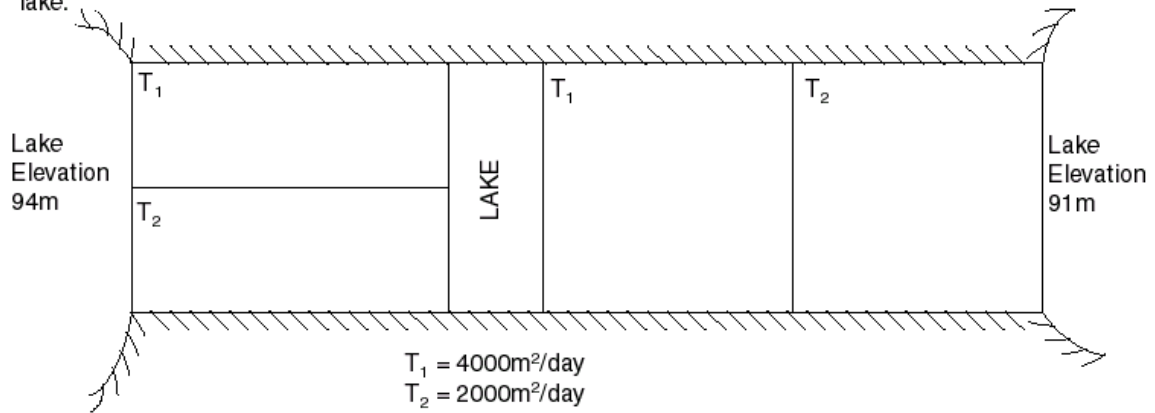
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Question 9:

A. Construct a flow net for the following diagram.

B. Calculate the discharge from one lake to the other.

C. Determine the elevation of the water surface in the small rectangular lake.



Week 2 - Numerical Models

For this assignment, you will prepare and run models of four different problems. The first two are systems for which you prepared flownets last week. The other two are more complicated problems.

Problem 1:

Prepare a MODFLOW model of Question 5 from last week. Discuss how your flownet and model results are similar and/or different. Which method do you think is better? Why? Include a copy of your output and grid design.

Directions for making and running the model:

1. Open Visual MODFLOW.
2. Create a new model:
 - i) Choose: **File>New** goto the directory "...HWR431LAB5/PROBLEM1"
Name the model "*Problem1.vmf*"
 - ii) The *Unit Selection* screen will open. Choose the following:

<i>Length Unit:</i>	<i>meters</i>
<i>Conductivity Unit:</i>	<i>m/day</i>
<i>Time Unit:</i>	<i>days</i>
<i>Pumping Rate:</i>	<i>m³/day</i>
<i>Recharge:</i>	<i>mm/year</i>

(NOTE: these are the defaults except for the conductivity unit)

The *Mass Units* screen will open. Keep the defaults.
 - iii) Retrieve site coordinates from a DXF MAP.
Choose the file: "...Problem1/Sitemap1.dxf"
 - iv) *Mesh Dimensions:*

Number of columns:	67
Min. X:	0
Max. X:	10
Number of rows:	40
Min. Y:	0
Max. Y:	6
Number of layers:	1
Min Z:	0
Max. Z:	1
 - v) Define the model space:
From the menu bar choose **Input**
From the button bar choose **Inactive Cells> Mark Single**
Colour in the cells outside of the flow domain (you can also use the *Mark Polygon* tool to do this and then fill in details with the *Mark Single* tool)
NOTE: Do not fill in the cells on the right hand side of the model where the constant head boundary will be.
 - vi) Set the constant head boundaries:
From the menu select **Boundaries > Constant Head**
Select **Assign Single** from the button bar.
Fill in the cells which will be represent the constant head boundary on the right hand side of the model domain. These cells should be highlighted in pink. Before selecting "OK" set the following:

Stop Time:	1 day
Constant Head:	217m

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Press "OK". The cells should now be burgundy.

Assign the other constant head boundary (i.e. the lake) using the same method but making the value of the head here 218m.

- vii) Set the aquifer properties:
From the menu bar choose: **Properties > Conductivity**
Set the *Default Property Values*:
- | | |
|---------------|------|
| $K_x = K_y$: | 4800 |
| K_z : | 0 |
| S_s | 0 |
| S_y | 0.23 |
| Eff.Por: | 0.23 |
| Tot.Por: | 0.23 |

NOTES: *The data we have for the assignment is transmissivity, not conductivity. However, since $T=Kb$ and we initially set $b=1.0m$ (i.e. uniform aquifer thickness of 1m) for this case we have $T=K$.*

Also, note that since we are using a steady state model and are not interested in particle velocities, our selection of the storage parameters and porosity is arbitrary.

The Default Property Values dialog sets the properties for all cells. In problem 2 you will modify individual cells to create a heterogeneous system.

3. Run your model.
 - a. Go to the main menu: **File > Main Menu**
 - b. Go to the runtime options: **RUN**
Select *STEADY STATE* for the model Run Type
 - c. From the menu bar choose **RUN**
Select *MODFLOW* as the model
The model will run for a few minutes. As it does so, it will display a graph showing progress towards convergence. Your models should converge within about 100 iterations. If they do not, check your input to make sure you did not make a mistake.
After the model stops running, a blue checkmark will appear in the dialog. Select **EXIT**.
4. Check your results.
 - a. From the menu bar select **Output**.
 - b. Play with the output options. Make especially sure to try the *inspect cell* button on the tool bar, the *overlay options* on the toolbar at the bottom of the screen, and the options you get when you press the right mouse button.
 - c. Make a printout of the head distribution you obtained. (NOTE: You can print to a .dxf file which can be imported directly as a picture to MS Word97.)

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Problem 2:

Prepare a MODFLOW model of Question 7 from last week. Discuss how your flownet and model results are similar and/or different. In the output screen, change the overlay settings for the equipotentials such that the interval is 3.333m. How does this diagram compare with your flownet from last week? Comment on how the size of the grid may impact your results. Include a copy of your output and grid design.

Directions for making and running the model:

Follow directions 1-7 from above with the following exceptions.

- i. Use a new directory for the new model and name the model "*Problem2.vmf*"
- ii. Do not import a .dxf file. Instead, make the grid 14x6. Make column 1 and 14 constant head boundaries.
- iii. When setting properties, make the default conductivity 300m/d. Then choose the assign single button. Choose to make a new property from the dialog box. Highlight the proper cells, THEN press ok on the dialog box. Half of the grid should now be blue.

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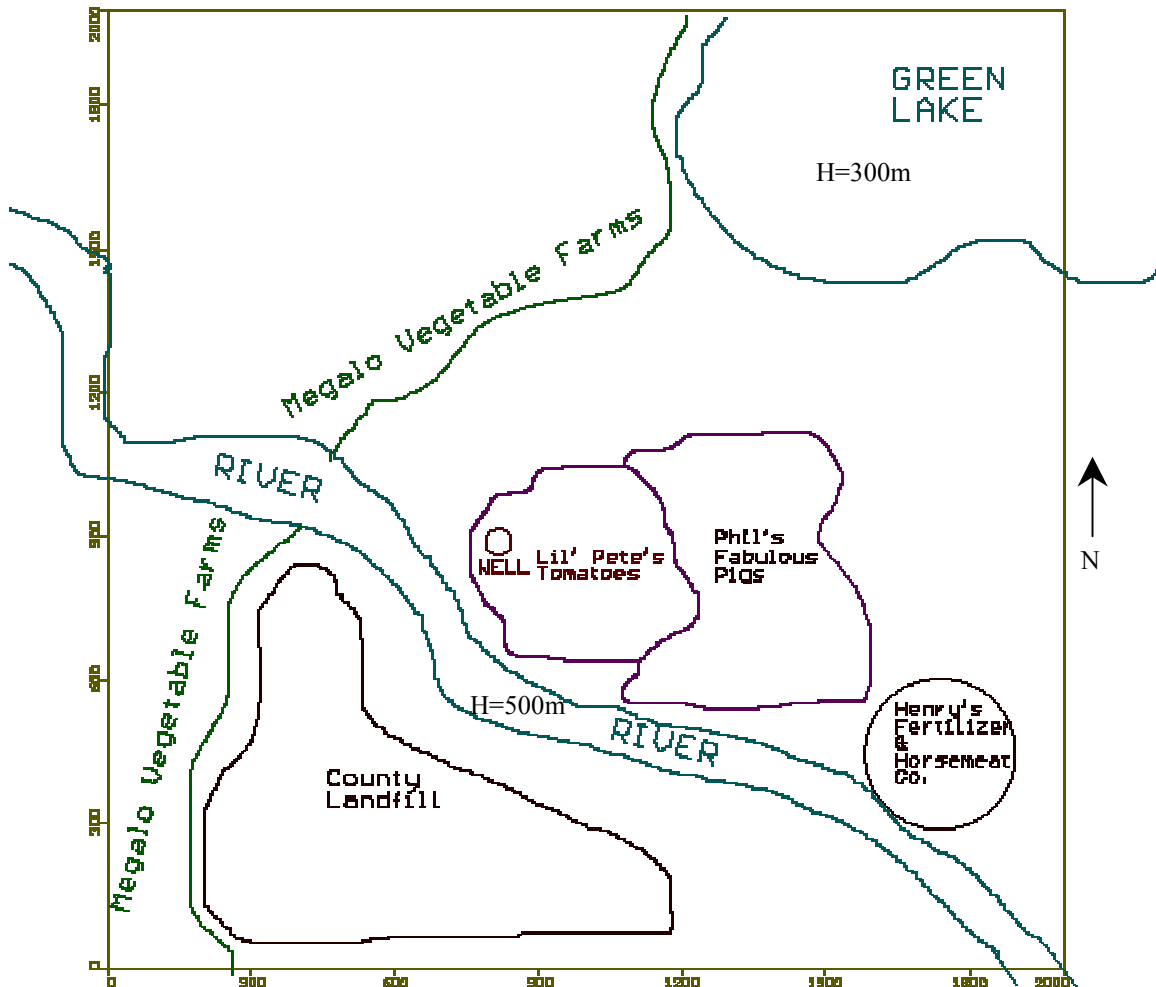
Problem 3:

Trouble is a brewin' in the valley. Lil' Pete is darn tooting mad 'cause he thinks his well water is being contaminated by Phil's-not-so-fabulous pigs. He is getting ready to sue Phil to cover the losses he faced on last year's bumper crop of mutant tomatoes and he wants to get the facts right before he hires Johnnie Cochran and turns this thing into a national scandal. He hires you to model the groundwater hydrology of the basin to corroborate his theory. Don't forget the other characters in this story. Could the culprit really be Megalo Vegetable Farms (you've heard that they are not organic farmers as they claim)? Maybe it is dear sweet Henry and his Fertilizer & Horsemeat Co. (you've heard that he dumps assorted horse parts and effluent into the river)! Perhaps it is a government conspiracy perpetrated by the County Landfill?

You are to prepare a 1 page report on your findings. You should discuss your decisions for preparing the model the way you did and outline your findings with regards to responsibility for the contamination.

You have the following information about the basin:

- The head in the lake is 300m
- The head in the river is 500m
- The head in the south-west corner of the grid is 250m
- The basin has a uniform transmissivity of 200m/day.
- The river runs west to east
- The boundaries of the basin where head is not yet specified should be constant head boundaries which change in a linear fashion between known boundaries (use the assign line tool in VMODFLOW)
- The area you should model is 2km x 2km.



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Problem 4:

Farmer Buck wants to install a new pump to aid in the irrigation of his crop of... (well, we couldn't tell you that or Farmer Buck would be in a lot of trouble!). Buck's neighbour Willie doesn't want the new well to interfere with his own well's production. Willie has said that he doesn't care how much Buck pumps as long it doesn't draw down the existing well by more than 0.5m. It is up to you to maximize Buck's pumping rate without making Willie go dry. Write a short report that describes how you set up your model and discusses your results.

(Hint: Make sure you establish the equilibrium baseline flow before you turn on Buck's well!)

You know the following about the area:

- There are 3 constant head boundaries and 3 impermeable boundaries
- The basin is heterogeneous with 5 different units - each unit is homogenous
- The area you should model is 2km x 2km.

