PART 5 Hydraulic properties of porous media

Porosity

Definition: $n = V_{void}/V_{total}$ total porosity

Void space: $e = V_{void}/V_{solid}$

Primary porosity - between grains

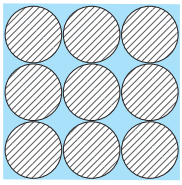
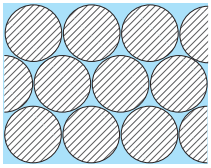
Secondary porosity - fracture or solution porosity

Primary porosity

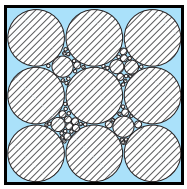
Uniform spheres - porosity constant, independent of particle size

Can have different packing:

Table 6. Different geometric arrangements of uniform cubic spheres.

1	2	3-5	6
		other configurations	same as # 2, but in 3-D
cubic packing			rhomboedral packing
n = 0.476	0.476 > n > 0.259	0.476 > n > 0.259	n = 0.259
least compact			most compact

Non-uniform grains will have lower porosity because small grains fill void spaces between larger clasts.



Spheres can form “stable bridges” -> porosity up to 0.7

Secondary porosity

Due to postdepositional processes, such as dissolution and re-precipitation, fracturing, etc.

Overhead from Carozzi: p. 11 and p. 25 (both on one page)

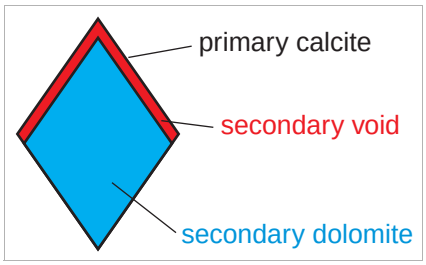
Mineral replacement, e.g.,

$2CaCO_3 \rightarrow (Ca,Mg)CO_3$

calcite dolomite

2.71 g/cm^3 2.87 g/cm^3

5.9% reduction of volume



Another example:

gypsum $\rightarrow$ anhydrite

2.32 g/cm^3 2.98 g/cm^3

28% reduction of volume

Overhead from Carozzi, p. 141

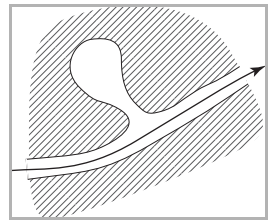
Fractures - similar definition of porosity: V_{void}/V_{total}

Types of porosity

Total porosity - defined earlier; total pore space in a porous medium; total water content

$$n = \frac{\text{pore volume}}{\text{total volume}}$$

Effective porosity - interconnected pore space. Some water is in dead-end pores or otherwise captured and excluded from circulation.



$$n_e = \frac{\text{interconnected pore volume}}{\text{total volume}}$$

Kinematic porosity - mobile water in interconnected pore space.

$$n_e = \frac{\text{volume of water able to circulate}}{\text{total volume}}$$

From the point of view of fluid flow, adhesive water acts as solid. The volume of water that circulates is smaller than the water in interconnected pore space.

We can assume that the following is true:

total porosity > effective porosity > kinematic porosity

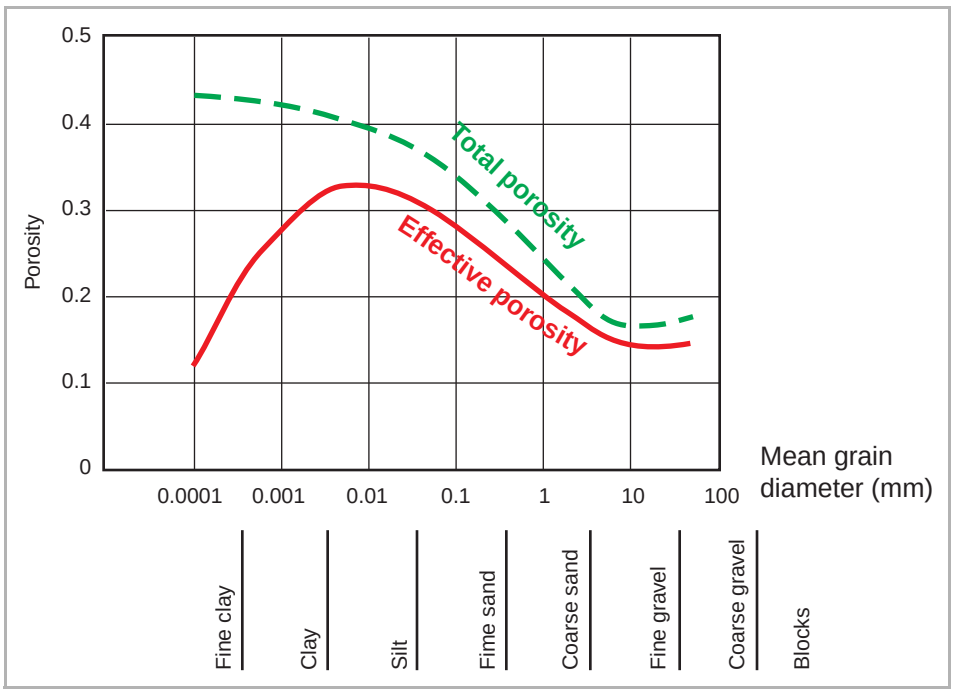
Kinematic implies fluid in motion, effective - not necessarily in motion. However, often, effective and kinematic porosities are considered equivalent.

Values of porosity

Table 7. Typical values of porosity and effective porosity for common geological materials.

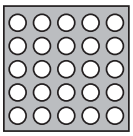
Material	Total porosity, % (mostly from deMarsily, 1986)	Effective porosity, % (from Domenico and Schwartz, 1990)
Granite, gneiss	0.02 - 1.8	0.0005
Basalt	3 - 35	
Weathered granite	34 - 57	
Volcanic tuff	30 - 40	
Sand	15 - 48	
Sandstone	4 - 38	0.5 - 10
Quartzite	0.8	
Clay	44 - 53	
Swelling clay	up to 90	
Shale	0.5 - 7.5	0.5 - 5
Limestone (primary porosity)	0.5 - 12.5	0.1 - 5
Dolomite (secondary porosity)	10 - 30	

In general terms, porosity and effective porosity are related as in the figure below:



Intrinsic permeability

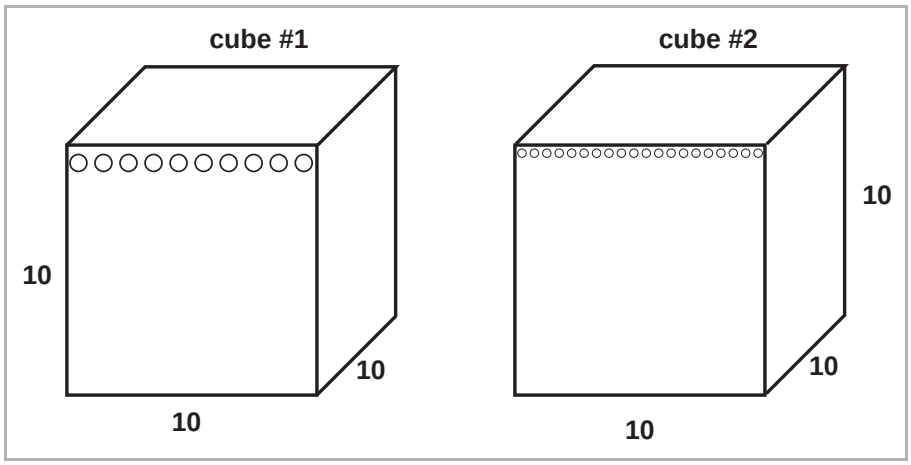
Look again at total flux through our porous medium composed of tubes:



$$Q_T = -B\frac{na^2}{8\mu}\cdot\frac{\partial p}{\partial x}$$

We saw that effective (or kinematic) porosity affects Q_T. A second, more important factor is tube diameter 2a, or grain diameter, which determines the **intrinsic permeability** of porous media.

Consider two cubes 10×10×10 mm with tubes of two sizes: (1) a=0.5 mm and (2) a=0.25 mm.



Porosity remains the same, at 0.785 (assuming that tube walls are very thin). Let $\mu = 10^{-3}$ Pa s and $\partial p/\partial x = 1$. Calculate Q_T:

$$Q_T = -B\frac{na^2}{8\mu}\cdot\frac{\partial p}{\partial x} = 9817.5a^2$$

Cube 1: $Q_T = 2454\ mm^3\ s^{-1} = 0.21\ m^3\ d^{-1}$

Cube 1: $Q_T = 614\ mm^3\ s^{-1} = 0.05\ m^3\ d^{-1}$

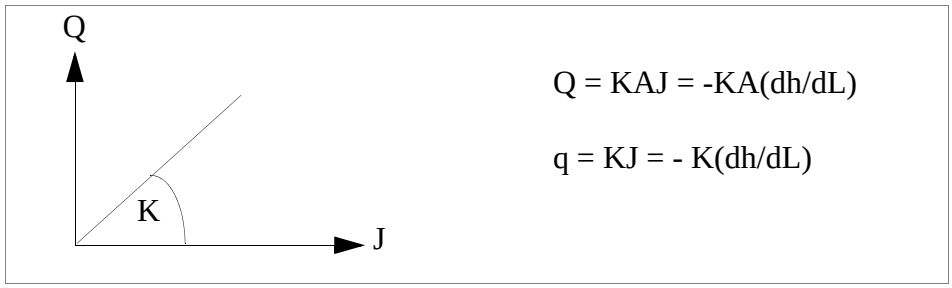
For $a=10^{-3}$ mm (clay fraction), $Q_T = 0.00245\ mm^3\ s^{-1}$ (over area 1×1 cm)

Consider this flow rate on a scale of aquifer, say 10 m thick, per 1 m width of aquifer. The cross sectional area is 10 m^2, i.e., $10^7\ mm^2$. $Q_T = 245\ mm^3\ s^{-1} = 0.021\ m^3\ d^{-1}$. So, we need 47 days to supply 1 m^3 of water.

Is this a good aquifer?

Hydraulic conductivity

In Darcy's law, the proportionality constant K is called hydraulic conductivity (units: L T^{-1}). Graphically, it is the slope of the line in Darcy's law:



What does the hydraulic conductivity depend on?

Recall flow through a tube. Discharge per unit area (specific discharge) is:

$$q = \frac{-a^2}{8\mu} \cdot \frac{\partial p}{\partial x} \qquad \textit{and} \quad p = \rho g h$$

$$q = \frac{-a^2}{8\mu} \cdot \frac{\partial(\rho g h)}{\partial x} \qquad \rho \text{ and } g = \text{const}$$

$$q = \frac{-a^2 \rho g}{8\mu} \cdot \frac{\partial h}{\partial x} \qquad \rho \text{ and } g = \text{const}$$

so, by analogy with Darcy's law (q = -K dh/dL)

$$K = \frac{a^2 \rho g}{8\mu} \qquad \text{property of medium (1/8 and } a^2\text{) and fluid } (\rho \text{ and } \mu)$$

Dimension of K:

$$\frac{L^2 M L^{-3} L^2 T^{-2}}{M L^2 T^{-2} L^{-2} T} = L T^{-1} \qquad \text{K has units of velocity}$$

Separate factors in K into two groups:

$$K = \frac{a^2 \rho g}{8\mu} = \frac{a^2}{8} \cdot \frac{\rho g}{\mu} = k \cdot \frac{\rho g}{\mu}$$

Term $a^2/8$ (k, intrinsic permeability), is a function of the porous medium alone. In general, it is considered proportional to some characteristic length, e.g., grain size: $k = cd^2$, where c is a dimensionless proportionality constant that may be found experimentally, d is median grain size (d_{50}) or the tenth percentile (d_{10}) or some other length related to grain size.

Dimension of k: L^2

Common unit: **darcy** $=10^{-8}$ cm^2

<u>Typical values of hydraulic conductivity (m/s):</u>

gravel	10^{-3} - 10^{1}	clay	10^{-12} - 10^{-5}
sand	10^{-7} - 10^{-2}	karst limestone	10^{-5} - 10^{-1}
sandstone	10^{-10} - 10^{-5}	crystalline rock	10^{-13} - 10^{-10}
silt	10^{-9} - 10^{-5}		

Study Table 2.2. in Freeze & Cherry: you should know the range of values of hydraulic conductivity (K) and intrinsic permeability (k) for different media

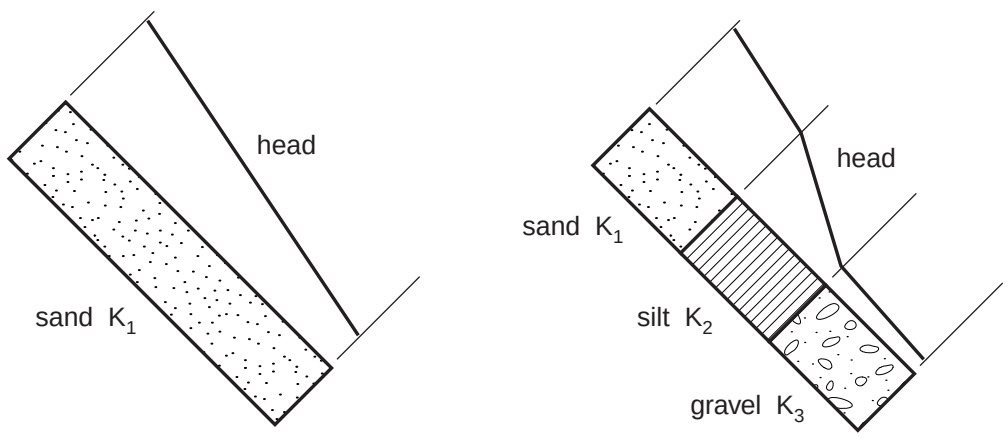
Heterogeneity of hydraulic conductivity

Properties vary in space -> **heterogeneous**

Steady properties -> **homogeneous**

Examples of soil heterogeneity:
- clay lenses in sand formation
- fractures in crystalline rocks

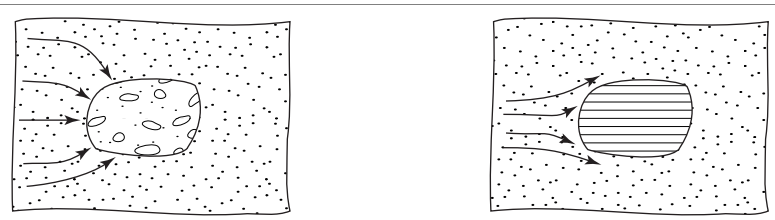
Consider two laboratory columns: one is packed with the same soil, the other contains three soils with different hydraulic conductivities. We can deduce the distribution of hydraulic head from Darcy's law (q = -K dh/dL), assuming that the discharge q is constant throughout the column.



Linear head drop along entire length of column

Linear head drop only locally, within single unit. Most energy dissipated in silt.

Real aquifer examples: lenses of gravel (left panel) and clay (right panel) in uniform sand.

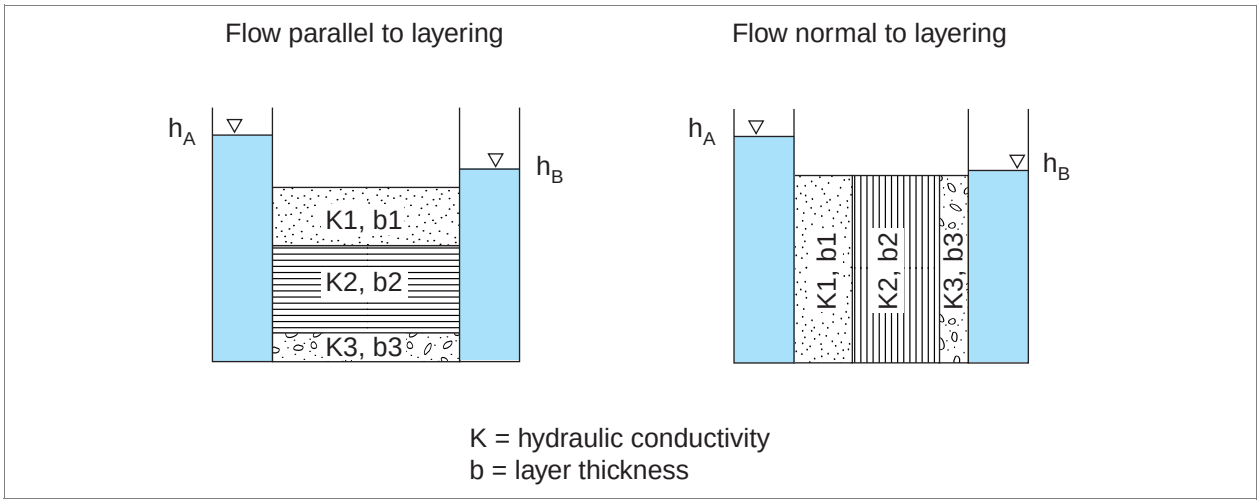


Water flows through more permeable localities.

Note, however, that it applies only in situations where water has a choice of direction, that is, where there are few constraints on the direction of flow. We will see later that water will flow from a sand layer into a clay layer.

Effective hydraulic conductivity

Consider two simple cases of flow in one dimension (1-D flow):



We can calculate equivalent (or effective) hydraulic conductivity to represent the heterogeneous formation.

For flow parallel to layers, effective K is called K_h (for horizontal flow, because parallel flow is typically found in a horizontal layered sedimentary sequences).

$$K_h = \frac{\sum_{i=1}^{n} K_i b_i}{\sum_{i=1}^{n} b_i}$$

arithmetic mean

For flow normal to layers, effective K is called K_v (for vertical flow, because normal flow is typically vertical, across horizontal layers).

$$K_v = \frac{\sum_{i=1}^{n} b_i}{\sum_{i=1}^{n} \frac{b_i}{K_i}}$$

harmonic mean

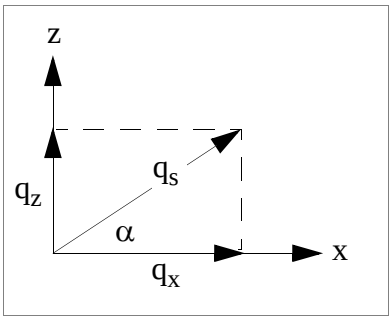
Effective hydraulic conductivity must preserve the flux Q (or q), but not the head.

In real aquifers, the flow is at an angle to the bedding (layering) and the best effective hydraulic conductivity is calculated as geometric mean: K_G = ?

In a layered formation, effective K is different in horizontal and vertical directions. By using this averaging process, we replace many **isotropic** units with a single **anisotropic** medium.

Anisotropy of hydraulic conductivity

Anisotropic properties of aquifers can be described by giving K values in all directions. Consider an anisotropic medium in which flow q_s is along direction s.



Flux q_s can be decomposed into its components q_x and q_z along axes x and z:

$q_x = q_s \cos\alpha = -K_x\, \partial h/\partial x$

$q_z = q_s \sin\alpha = -K_z\, \partial h/\partial z$

In the expressions above, K_x and K_z are hydraulic conductivities in the x and z directions.

The flux along the direction s is: $q_s = -K_s\, \partial h/\partial s$

Look at term $\partial h/\partial s$ using the chain rule:

$$\frac{\partial h}{\partial s} = \frac{\partial h}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial h}{\partial z} \cdot \frac{\partial z}{\partial s}$$

but we know that

$$\frac{\partial x}{\partial s} = \cos\alpha \quad \text{and} \quad \frac{\partial z}{\partial s} = \sin\alpha$$

that

$$\frac{\partial h}{\partial s} = -\frac{q_s}{K_s}, \quad \frac{\partial h}{\partial x} = -\frac{q_x}{K_x} \quad \text{and} \quad \frac{\partial h}{\partial z} = -\frac{q_z}{K_z}$$

and that

$$-\frac{q_s}{K_s} = -\frac{q_x}{K_x}\cos\alpha - \frac{q_z}{K_z}\sin\alpha$$

Because $q_x = q_s \cos\alpha$ and $q_z = q_s \sin\alpha$, we get:

$$-\frac{q_s}{K_s} = -\frac{q_s}{K_x}(\cos\alpha)^2 - \frac{q_s}{K_z}(\sin\alpha)^2$$

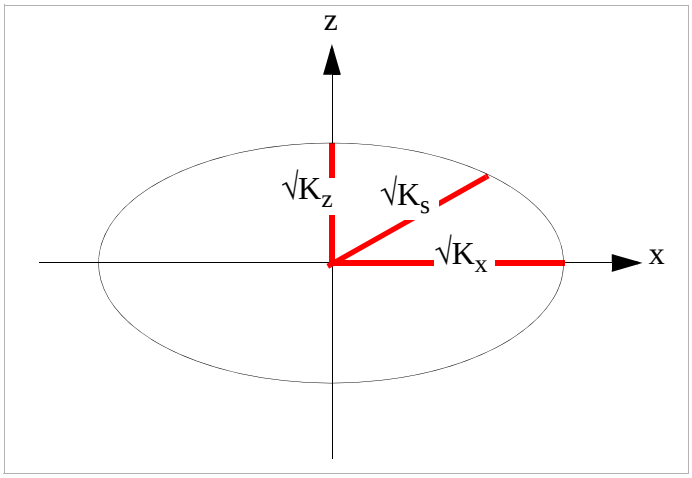
Simplifying, we get:

$$\frac{1}{K_s} = \frac{(\cos\alpha)^2}{K_x} + \frac{(\sin\alpha)^2}{K_z}$$

or, substituting x = r cosα and z = r sinα

$$\frac{r^2}{K_s} = \frac{x^2}{K_x} + \frac{z^2}{K_z}$$

Given K_x and K_z, we can use this formula to calculate K_s in any direction determined by angle α. Graphically, these K values are related as follows (thick red lines):



hydraulic conductivity ellipse (2-D)

In Cartesian coordinate system, in 3-D, it is an "ellipsoid":

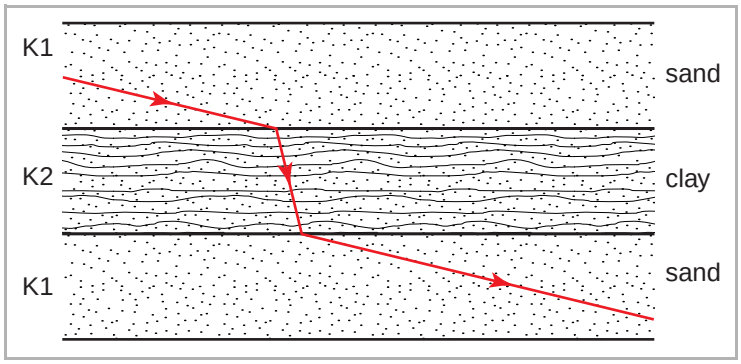
$$\frac{r^2}{K_s} = \frac{x^2}{K_x} + \frac{y^2}{K_y} + \frac{z^2}{K_z}$$

hydraulic conductivity ellipsoid (3-D)

Major axes are: $\sqrt{K_x}$, $\sqrt{K_y}$ and $\sqrt{K_z}$.

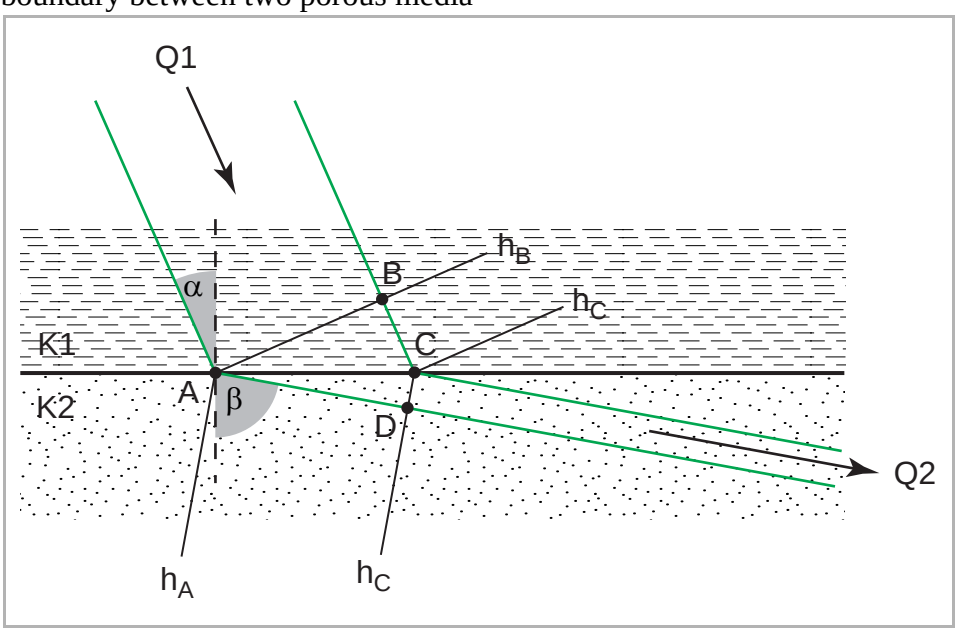
Boundaries between layers

If flow is not parallel to layers and not normal, we observe refraction of flow lines because of anisotropy.



In the system above, K1 is much greater than K2. This contrast controls refraction of flow lines: the greater the contrast between K values, the stronger the refraction. We will develop this relation quantitatively.

Consider a boundary between two porous media



From Darcy's law, we calculate fluxes (per unit thickness):

$$Q_1 = -K_1 \frac{h_C - h_A}{|BC|} |AB|$$

$$Q_2 = -K_2 \frac{h_C - h_A}{|AD|} |CD|$$

where absolute values represent lengths (for example, |AB| is the distance from A to B).

Because the flow is constant (or steady state), the fluxes must be equal, $Q_1 = Q_2$

$$-K_1 \frac{h_C - h_A}{|BC|}|AB| = -K_2 \frac{h_C - h_A}{|AD|}|CD|$$

$$\frac{-K_1}{|BC|}|AB| = \frac{-K_2}{|AD|}|CD|$$

$$\frac{K_1}{K_2} = \frac{|BC||CD|}{|AB||AD|} = \frac{\tan\alpha}{\tan\beta}$$

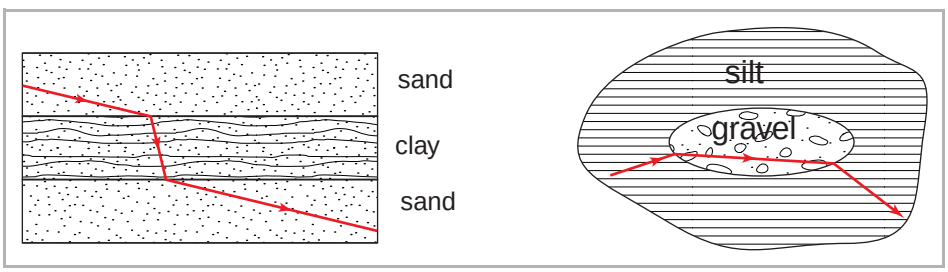
$$\boxed{\frac{K_1}{K_2} = \frac{\tan\alpha}{\tan\beta}}$$

Tangent law for refraction of ground-water flow lines at geologic boundaries

<u>Example</u>

* If $K_2 >> K_1$, then $\tan\beta >> \tan\alpha$, and $\beta \to 90°$, i.e., flow is nearly parallel to the boundary

* If $K_2 << K_1$, then $\tan\beta << \tan\alpha$, and $\beta \to 0°$, i.e., flow is nearly normal to the boundary



Heterogeneity and anisotropy

Scalar properties:

Homogeneity - property is constant in space

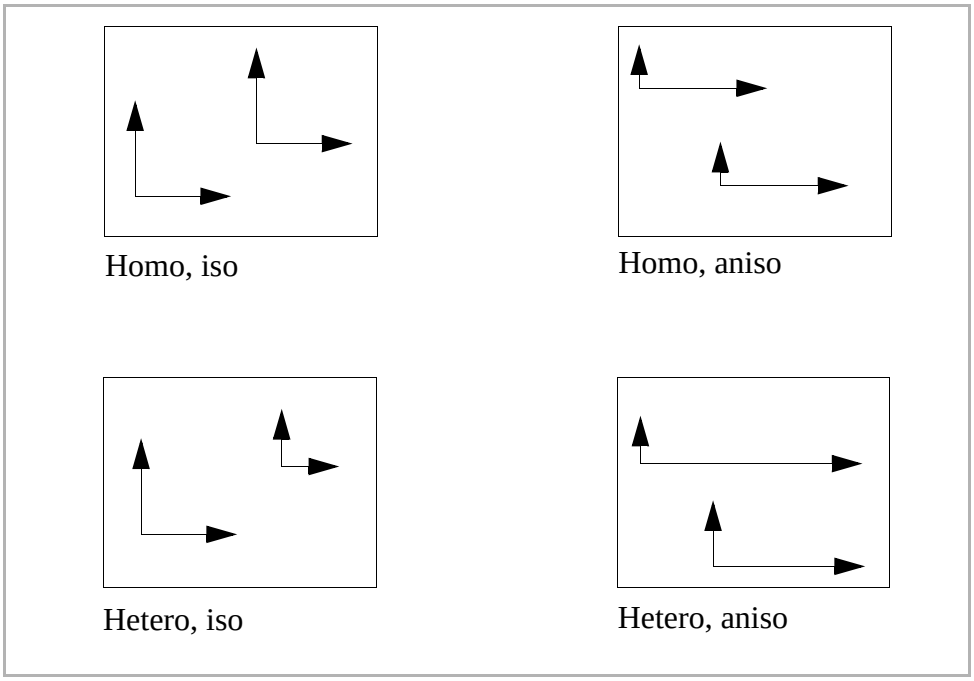
Heterogeneity - property varies in space

Vector properties (tensor):

Isotropy - property invariant with direction

Anisotropy - property varies with direction

There are four possible combinations of heterogeneity and anisotropy:

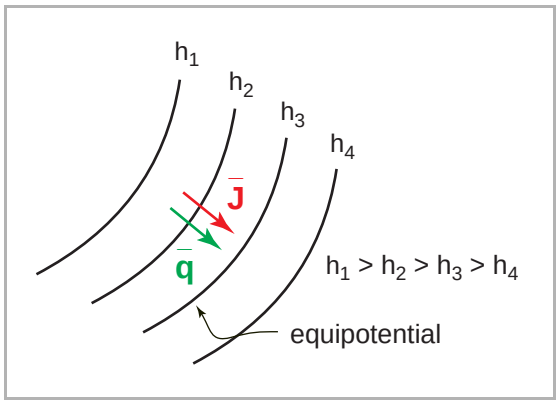


Influence of K on the direction of flow

(1) Homogeneous and isotropic aquifer

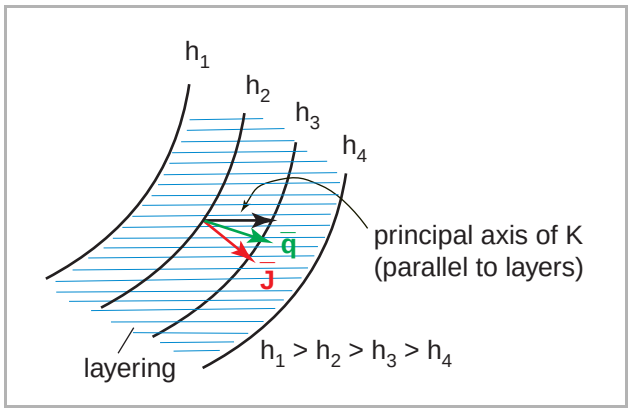
The direction of specific discharge $\bar{q}$ is normal to equipotentials (lines of equal potential or equal head).

Vectors of head gradient $\bar{J}$ and discharge $\bar{q}$ are in the same direction.

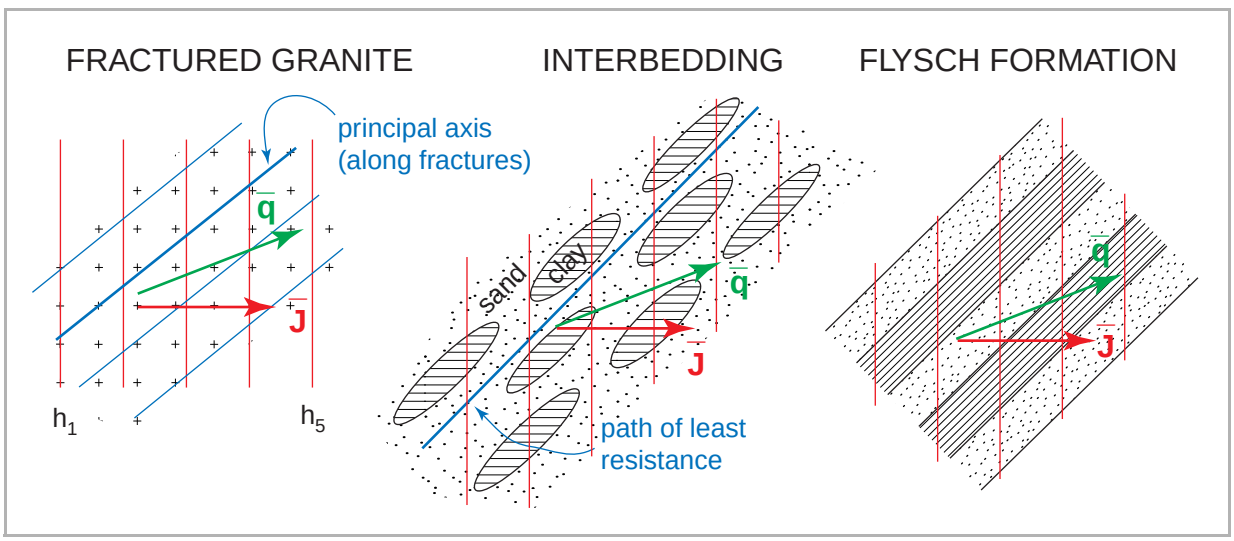


(2) Homogeneous and anisotropic

Vector $\bar{J}$ is as before, but the direction of $\bar{q}$ is somewhere between $\bar{J}$ and the direction of layering (here also the path of least resistance).



<u>Examples</u>



Hydraulic conductivity tensor

Hydraulic conductivity has directional properties. It is described by second order symmetric tensor (K with double overbar; twin overbar denotes tensor):

$$\overline{\overline{K}} = \begin{bmatrix} K_{xx} & K_{xy} & K_{xz} \\ K_{yx} & K_{yy} & K_{yz} \\ K_{zx} & K_{zy} & K_{zz} \end{bmatrix}$$

where entry K_{ij} is the specific discharge in the i direction due to a unit gradient in the j direction.

Anisotropy is a matter of scale: small scale -> isotropic; aquifer scale -> anisotropic.

In two dimensions (here, x and y), the hydraulic conductivity tensor is:

$$\overline{\overline{K_{xy}}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}$$

Example:

The hydraulic conductivity is described by tensor:

$$\overline{\overline{K_{xy}}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.3 \end{bmatrix} \text{ m d}^{-1}$$

which means the following:

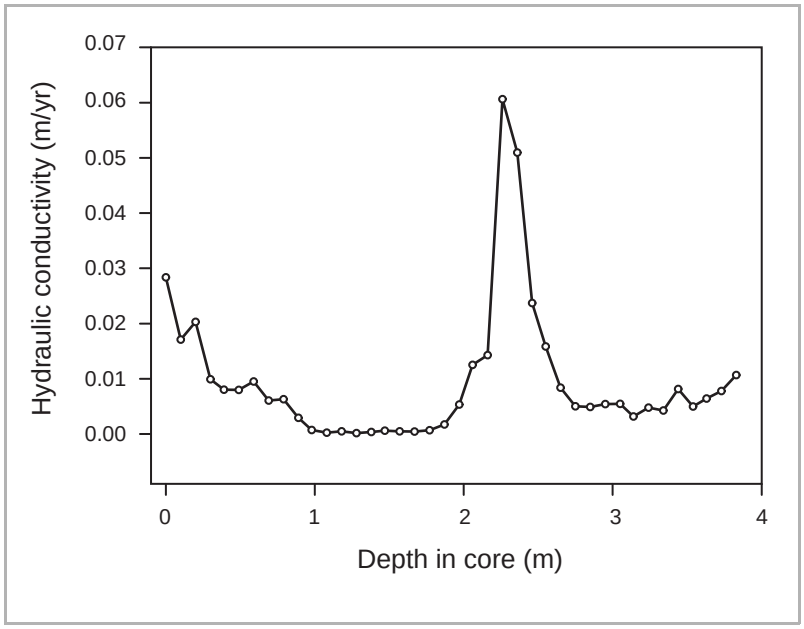
(1) under unit gradient in the x direction, the specific discharge in the x direction is 0.5 (K_{xx})
(2) under unit gradient in the y direction, the specific discharge in the x direction is 0.1 (K_{xy})
(3) under unit gradient in the x direction, the specific discharge in the y direction is 0.1 (K_{yx})
(4) under unit gradient in the y direction, the specific discharge in the y direction is 0.3 (K_{yy})

Description of soil heterogeneity

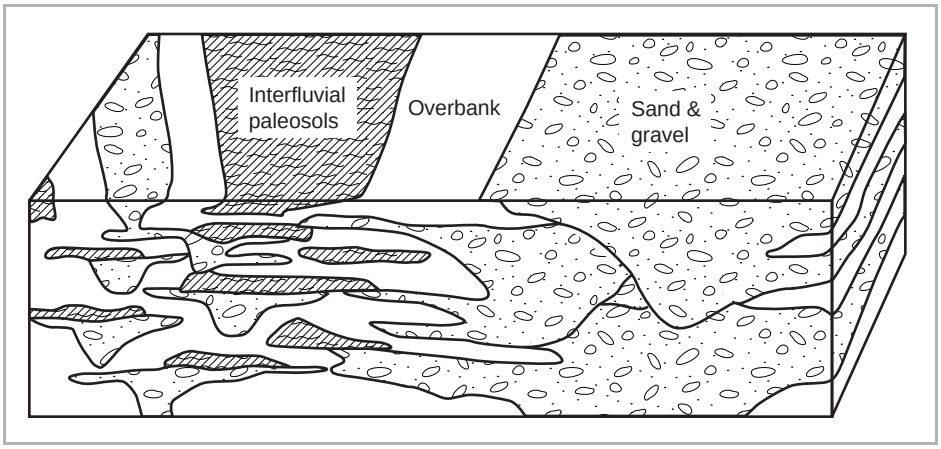
Look at the example of the distribution of hydraulic conductivity shown in the figure (to the right). Observe that high values of K are accompanied by other high values, and low values by other lows. We say that similar values of K are clustered together, i.e., that **K is spatially correlated**.

What are possible reasons for such a structure?

Look at the figure below.



Go back to homework problem in which we looked at hydraulic conductivity in a core. Look at assumptions that we made about K outside the core. We assume that K outside the core is the same as in the core. Is this a valid assumption? Look at the figure to the right.



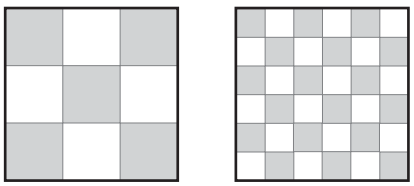
Questions:

(1) Is effective K really that effective?

(2) Can we calculate the same effective K for two completely different systems?

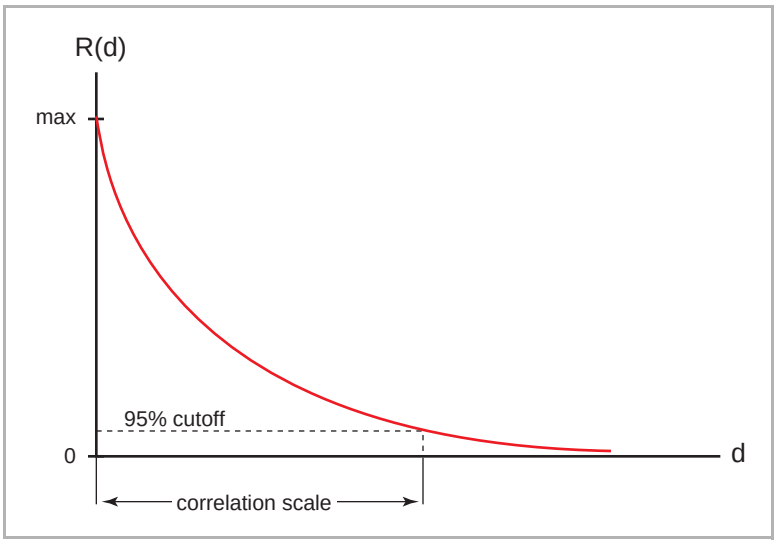
(3) What is missing in our effective K calculations?

Two ways to describe heterogeneity

(1) Deterministic	**(2) Statistical**
Effective K in different directions. Does not take into account correlation between K values. The two systems below would produce the same effective K. 	Looks at spatial statistical structure of K values. Let us predict the K value at location (x+1) given the K value at location x. We can assign some probability to our prediction. Gain over deterministic way: we look at values of K and the way these values change in space. Correlation scale - concept.

Correlation scale - distance over which values of K are strongly correlated.

Covariance - determines the structure of the spatial changes of K. It is a function of distance (d) between two points of measurements (see figure below).



R(d) = covariance

d = distance

Computational formula for R:

$$R = \frac{1}{N-1} \cdot \sum_{i=1}^{N} (x_{ij} - \bar{x}_j) \cdot (x_{ik} - \bar{x}_k)$$

where N is the number of pairs, x is the value and $\bar{x}$ is the average value.